ABSTRACT

DIVIDING SPACES: A HISTORICAL, PHILOSOPHICAL, AND THEORETICAL APPROACH TO INCOMMENSURABLE INTERVAL RATIOS

By

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This study explores the idea of incommensurability and its relation to the original composition *Anomaly* for solo flute with electronics and ensemble. Incommensurability is the point of tension and the cognitive gap between two distinctly different ways of viewing the world. This concept is closely associated with Pythagorean musical theory, particularly with the geometric mean and the irrationality of the square root of two. After arguing for and demonstrating the application of the geometric mean, this study relates *Anomaly* to concepts of overtone and time in Gérard Grisey's spectral masterpiece *Les éspaces acoustiques*.

DIVIDING SPACES: A HISTORICAL, PHILOSOPHICAL, AND THEORETICAL APPROACH TO INCOMMENSURABLE INTERVAL RATIOS

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INTRODUCTION

This study explores the idea of incommensurability through its multifarious relation to ancient Greek culture, mathematical concepts and musical theories, and demonstrates how incommensurability applies towards interval division in my composition *Anomaly*, for solo flute with electronics and ensemble. The problem of incommensurability originated in ancient Greece, specifically in the number-cult of Pythagoras. The term itself refers to the discovery of the first irrational number, the square root of two, in a world that knew only whole numbers. Incommensurability is the point of tension and the cognitive gap between two distinctly different ways of viewing the world. In what comes down to us through history as both math and myth, incommensurability is at once revolutionary, controversial, and troubling; not the least in part in that it concerns an alleged murder and cover up.

More recently, philosopher Thomas Kuhn borrowed the idea of incommensurability when speaking of historical scientific discoveries: "The normal-scientific tradition that emerges from a scientific revolution is not only incompatible but often actually incommensurable with that which has gone before."[1] In ancient Greece, the paradigm of mathematical thought that included irrational numbers were

1. Thomas Kuhn, *The Structure of Scientific Revolutions* (Chicago: University of Chicago Press, 1996), 103.

fundamentally in conflict with the paradigm that dismissed them. This conflict centered on the discovery of the first irrational number, an event that defied explanation by one paradigm, and provided the premise for the other paradigm's existence. Such events that violate the expected and ultimately cause the viewpoint of a group of people to shift are called anomalies.

Apart from the legend and the philosophy, the mathematical implication of the incommensurability of the square root of two is inextricably linked with the search for a proper division of the octave. This paper will examine different methods proposed by the Pythagoreans to divide the octave, and will show how one of the options they discarded provides the impetus for my composition *Anomaly*. The purpose of presenting the subjects in this paper in relation to *Anomaly* is to demonstrate that the abandoned theoretical division can be musically viable.

To this end, I will present a thorough description of the concept and techniques used in composing *Anomaly* with a particular focus on the invented scales and intervals resulting from irrational number ratios. I have also chosen to align *Anomaly* with the spectral movement, associated with composers Gérard Grisey and Tristan Murail. The spectral focus on finding musical order in the nature of sound itself is closely tied to the Pythagorean understanding of music and number. I will draw comparison between *Anomaly* and parts of Grisey's spectral masterpiece *Les éspaces acoustiques*, concentrating on the use of overtones and the treatment of time elements in both pieces.

The conclusions drawn from this study converge on the theory of the division of the octave. The solutions to this problem developed in ancient Greece shaped the musical heritage of the West for the next two millennia. I will show how the techniques demonstrated in *Anomaly* explore an alternate paradigm to the age-old questions of dividing frequency space.

CHAPTER 2

INCOMMENSURABILITY IN ANCIENT GREECE

Math, Myth, and Mysticism

The origins of Western musical tradition and the problem of dividing intervallic spaces begins in ancient Greece with a group of thinkers associated with Pythagoras of Samos. While none of his writings remain, Pythagoras's teachings greatly influenced philosophic, religious, and mathematical thought beginning in the second half of the sixth century BCE.[1] The surviving sources that make reference to the Pythagorean traditions are fragments from Archytas, Plato's *Timaeus*, Aristotle's *Metaphysics,* Aristoxenus's *Elementa Harmonica*, the Euclidean *Sectio Canonis*, Nicomachus's *Enchiridion*, and Ptolemy's *Harmonics*. In and among the content from these sources we find mathematical observations and explanations, doubtful tales of inspiration and bloodshed, and the spiritualization of numbers and numerology. These writings also contain the first attempts at what can imperfectly be called music theory,[2] central to which is the division of string lengths on a monochord to produce intervals.

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1. André Barbera, "Pythagoras," in *The New Grove Dictionary of Music and Musicians*, 2nd ed. (London: Macmillan, 2001), 20:642.

2. Thomas J. Mathiesen et al., "Greece, Ancient," in *The New Grove Dictionary of Music and Musicians*, 2nd ed. (London: Macmillan, 2001), 10:335.

Following an overview of some important concepts relating to Pythagorean thought, this study will explore different solutions the ancient Greeks proposed for the division of the octave.

Background on the Pythagorean Cult

Given the silence of the master himself, the scarcity of writings by his early followers, and the secrecy of their movement, it is difficult to explain or even summarize the mid-sixth-century school of thought built around Pythagoras of Samos. What is factual about Pythagoras the man is difficult to ascertain given the lore that grew up around him in the centuries after his death. A philosopher and teacher, he fostered a school and religious sect that flourished for a while in Croton (in southern Italy) before being chased out of the city. Afterward, Pythagoras was said to have finished his days in Metapontum (on the southern coast of Italy).[3]

The cult built around Pythagoras was secretive and selective. Their beliefs and philosophical holdings generally concern observable science and mathematics with an emphasis on whole numbers. Pythagoras himself supposedly quipped the phrase, "All is number,"[4] and though scholars argue whether or not this is true, later Pythagoreans and thinkers influenced by him do, in fact, uphold a type of divinity associated with number. In a passage ascribed to Philolaus in the fourth century BCE, Philolaus confirms the centrality of number in Pythagorean thought when he says, "All things which can be

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3. Iamblicus, *Life of Pythagoras, or Pythagoric Life*, trans. Thomas Taylor (London: J. M. Watkins, 1818), 128.

4. Leonid Ja. Zhmud, "All is Number? Basic Doctrine of Pythagoreanism Reconsidered," *Phronesis* 34, no. 3 (1989): 270–1.

known have number; for it is not possible that without number anything can either be conceived or known."[5] Plato's famed creation of the soul of the universe in the *Timaeus* describes with esoteric detail the precise numerical relationships built into the universe.[6] Aristotle's mid-fourth-century BCE description of the Pythagoreans states that of their most important principles, "numbers are by nature the first, and in numbers they seemed to see many resemblances to the things that exist and come into being."[7] In Aristotle's view, the Pythagoreans interpreted the elemental truths of the natural world as related to numbers and even supposed that the heavenly realms were composed of musical scale and a number.[8]

To further explain the role of number in this view of the world, it is necessary to examine the Pythagorean understanding of proportion and ratio. Aristides Quintilianus, in a retrospective examination of the Pythagorean tradition, speaks of a creator:

> Call him Ratio or Unit, or, as men or divine wisdom have done, Unitary Ratio, revealing by the first title how he harmonizes and orders everything, and by the second how he has arrested the multiplicity and diversity of things, and holds them together in one with unbreakable bonds.[9]

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5. Ioannis Stobaei, *Eclogarum*, trans. August Meineke (Leipzig: Teubner, 1860), 1:21.

6. Plato, *Timaeus*, in *Source Readings in Music History*, rev. ed., ed. Oliver Strunk (New York: Norton, 1998), 20–3.

7. Aristotle, *Metaphysics*, trans. W. D. Ross (New York: Oxford University Press, 1924), 1:986a.

8. Ibid.

9. Aristides Quintilianus, *De Musica*, in *Greek Musical Writings*, ed. Andrew Barker (Cambridge: Cambridge University Press, 1989), 2:402.

The concept of ratio is viewed, at least by the late Pythagoreans, as being able to describe a type of divine ordering. These Pythagoreans extensively described and categorized ratios based on their numerical relationships. Ratios like 1:2, 1:3, and 1:4, for example, were called multiples, while ratios whose terms differed by one like 2:3, 3:4, and 4:5 were called epimores.[10] Assessments made about these ratios determined their beauty, perfection, and amount of reverence due.

One grouping of highly revered numbers among the Pythagoreans were the first four numbers in the integer series, otherwise known as the Tetrad.[11] They felt that these four numbers had mathematical properties unique among all integers. The first number added to the second produces the third ($1 + 2 = 3$). The second number added to itself produces its square ($2 + 2 = 2^2$). Finally, all four numbers added together result in 10, a type of unity in a base-10, or decimal, number system. The Tetrad also represented the organization of spatial dimensions from a point, to a line (the distance between two points), to a plane (the area between three points), to three-dimensional space (the volume between 4 points). Upon this basis, many formative mathematical and musical observations were made, chief among them being the Pythagorean Theorem.

<u>The Square Root of Two</u>

The Pythagorean Theorem is a mathematical description of the proportional relationships between the sides and the hypotenuse of a right triangle. The proof for this

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10. Richard L. Crocker, "Pythagorean Mathematics and Music (I)," *The Journal of Aesthetics and Art Criticism* 22, no. 2 (Winter 1963): 191.

11. André Barbera, "Arithmetic and Geometric Divisions of the Tetrachord," *Journal of Music Theory* 21, no. 2 (Fall 1977): 294.

involved calculating the area of squares made from each side of the triangle and comparing it to the square made by the hypotenuse (see Figure 1). These areas were shown to equal each other and, hence the formula $a^2 + b^2 = c^2$.

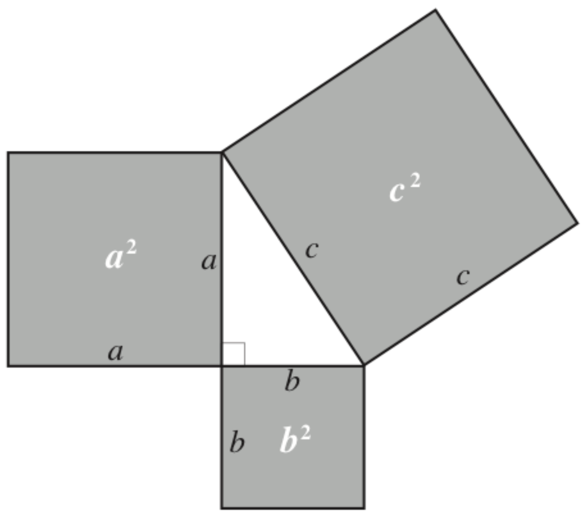

FIGURE 1. The proof of the Pythagorean Theorem.

Another unique property of the Tetrad, as put forth by the Pythagoreans, was that the last two numbers when squared and added together produced the square of the next number ($3^2 + 4^2 = 5^2$). This is the smallest possible integer example of the Pythagorean Theorem. Before long, it was discovered that the Theorem would not work for all numbers in the arena of arithmetic. A right triangle having sides a length of one would produce a hypotenuse whose length was the square root of two ($1^2 + 1^2 = x^2$; $x = \surd 2$). Because these early mathematicians had no concept for this type of number, the practice of that day would involve finding a larger number ratio to represent the fractional value.

For example, if the ratio was 1:1.5, the representational ratio could be enlarged and converted to 2:3. This, however, was soon found to be impossible with the square root of two. No ratio could represent its value, and it was therefore termed irrational, meaning without ratio.[12] Still, the fact of the matter remained that the hypotenuse was a discrete length that, if nothing else, could be represented visually. With the discovery of irrational numbers, the field of geometry was born.[13]

The discovery of the irrationality, or the incommensurability of the square root of two, expresses the fact that not every distance can be measured in whole numbers, or even fractions of whole numbers. It has been called by Kurt von Fritz, "one of the most amazing and far reaching accomplishments of early Greek mathematics."[14] This grates against a core philosophy of the early Pythagoreans, noted by historians who called its discovery a "*véritable scandale logique*."[15] The unfortunate man linked with this discovery is a Pythagorean philosopher named Hippasus of Metapontum. It is to his account that we must turn to next.

Hippasus of Metapontum

According to many of the earliest sources, Hippasus of Metapontum made the discovery of incommensurability. Hippasus was a contemporary and pupil of Pythagoras

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12. Thomas Heath, *A History of Greek Mathematics* (New York: Oxford University Press, 1921), 1:91.

13. Ibid., 90.

14. Kurt von Fritz, "The Discovery of Incommensurability by Hippasus of Metapontum," *The Annals of Mathematics* (2nd series) 46, no. 2 (April 1945): 242.

15. Paul Tannery, *Pour l'histoire de la science hellène* (Paris: Félix Alcan, 1887), 202.

himself. What little we know of him comes 800 years later in the writings of Iamblichus of Chalcis, a fourth-century neo-Platonist.[16] In Iamblichus, Hippasus is said to be "one of the Pythagoreans, but that in consequence of having divulged and described the method of forming a sphere from twelve pentagons, he perished in the sea, as an impious person."[17] Though he is speaking of another secret revealed, he later connects this with

> He who first divulged the theory of commensurable and incommensurable quantities, to those who were unworthy to receive it, was so hated by the Pythagoreans that they not only expelled him from their common association . . . but also constructed a tomb for him . . . he perished in the sea, as an impious person . . . who delivered the method of inscribing in a sphere the dodecædron.[18]

In the scholium on the beginning of Book 10 of Euclid's *Elements*, Proclus writes, "the first of the Pythagoreans who made public the investigation of these matters [that is, incommensurability] perished in a shipwreck."[19]

Modern authors have conjectured about the Pythagorean's anger for publically revealing the secret of incommensurability and the direct connection with a death at sea. Charles Seife gives an account that demonstrates the extent to which the myth has become sensationalized:

> Hippasus of Metapontum stood on the deck preparing to die. Around him stood the members of a cult, a secret brotherhood that he had betrayed. Hippasus had revealed a secret that was deadly to the Greek way of thinking, a secret that threatened to undermine the entire philosophy that the brotherhood had struggled to build. For revealing that secret, the great Pythagoras himself sentenced

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16. Iamblicus, i–ix.

17. Ibid., 47–8.

18. Ibid., 126.

19. Thomas Heath, *The Thirteen Books of Euclid's Elements* (Cambridge: Cambridge University Press, 1908), 3:1.

> Hippasus to death by drowning. To protect their number-philosophy, the cult would kill.[20]

While the repercussions of this revelation by Hippasus may come down to us exaggerated, or frankly untrue, what is true is that science and mathematics moved on. The discovery of the incommensurability of the square root of two may have challenged some sacred beliefs for the early Pythagoreans, but it eventually made possible the study of geometry, and in particular in the work of Euclid.

Application in Greek Music Theory

The importance of whole number ratios to the Pythagoreans in their understanding of philosophy, mathematics, and even the harmony of the universe itself is clear. However, the relationship of these ideas to music is not immediately apparent. These philosophers and scientists were not interested in musical discovery for its own sake, but only in its "paradigmatic and mimetic"[21] reflection of number. Ratios can represent intervals, and intervals can be objectively evaluated and classified based on their ratio. The two basic classifications are consonant intervals and dissonant intervals,[22] discussed in detail below. Finally, due to the reverence given the Tetrad, the Pythagoreans must have believed that the interval of an octave was intrinsically very

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20. Charles Seife, *Zero: The Biography of a Dangerous Idea* (New York: Penguin, 2000), 26.

21. Mathiesen, 10:335.

22. Euclid, *Sectio Canonis*, in *Greek Musical Writings*, ed. Andrew Barker (Cambridge: Cambridge University Press, 1989), 2:193.

valuable.[23] The pains to which efforts were made to find a suitable way to divide that space testifies to this fact. Following an overview of some important music theory concepts relating to Pythagorean thought, we will explore different solutions the ancient Greeks proposed for the division of the octave.

<u>Lengths of String</u>

The invention of the simple, single-string monochord used for scientific and teaching purposes is credited to Pythagoras.[24] A moveable bridge set beneath a fixed string allowed one to divide the string into two different lengths. The lengths could be equal, having a ratio of 1:1, resulting in the same pitch when plucked. A string twice as long as the other, having a ratio of 1:2, would create the interval of an octave. A ratio of 1:3 results in a twelfth, and 1:4 gives us the double octave. Figure 2 lists some simple ratios and their resulting intervals.

Ratio	Interval		Ratio	Interval
1:1	Unison		2:3	Perfect 5th
1:2	Octave		3:4	Perfect 4th
1:3	Perfect 12th		4:5	Major 3rd
1:4	Double Octave		5:6	Minor 3rd

FIGURE 2. A list of simple ratios with their corresponding musical intervals.

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23. Ptolemy, *Harmonics*, in *Greek Musical Writings*, ed. Andrew Barker (Cambridge: Cambridge University Press, 1989), 2:285.

24. Cecil Adkins, "Monochord," in *The New Grove Dictionary of Music and Musicians*, 2nd ed. (London: Macmillan, 2001), 17:2–4.

The methodology of these monochord experiments was unlike our understanding of the overtone series, which would have been foreign to the ancient Greeks: the smaller number in the ratio describes the shorter of the two string lengths, and thus the higher pitch. For example, in the ratio 2:3, the string length represented by 2 is the shorter, higher member of the perfect fifth.

Early Pythagoreans grouped intervals into a short list of consonant intervals, and a much larger list of dissonant intervals. This evaluation was based on their representative ratio and the inherent perfection or imperfection of the numbers involved. Included in the list of consonant intervals were the octave (1:2), the perfect twelfth (1:3), double octave (1:4), the perfect fifth (2:3), and the perfect fourth (3:4). Every other interval was relegated to the list of dissonant intervals.[25] The reason for this was sound: as explained above, the fascinating relationships found in the Tetrad, or the first four integers, fostered reverence for anything composed from their stock. The small, whole number ratios of these intervals bestowed on them the mark of perfection.

The octave, in particular, appears to have been held in quite high regard by Plato,[26] Nicomachus,[27] and Ptolemy.[28] Several reasons support this: Firstly, the octave's ratio consists of the first two numbers in the integer series. Outside of the unison, this

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25. Crocker, "Pythagorean Mathematics and Music (I)," 192.

26. Plato, 20–3.

27. Nicomachus of Gerasa, *Enchiridion*, in *Greek Musical Writings*, ed. Andrew Barker (Cambridge: Cambridge University Press, 1989), 2:261.

28. Ptolemy, 2:285.

makes it the first, and most perfect, interval. Ptolemy calls the octave, "the finest of the concords,"[29] that is, it sounds the most consonant compared to any other interval. In Plato's creation of the universal soul, the first act of the creator is to take, "one portion from the whole, and next a portion double of this,"[30] essentially describing a ratio of 1:2. The octave also has the special property of being formed by stacking two consonances: the perfect fifth (2:3) and the perfect fourth (3:4). Described in ratios, their combined ratio of 2:3:4 can be reduced to 1:2. This is unique in the integer series, being the only case where two consecutive epimore ratios (2:3 and 3:4) create the ratio directly before them (1:2).[31] It therefore makes sense that the Pythagoreans would use the octave as a measuring rod, a harmonic reference point, and an interval from which to generate new intervals.

<u>The Arithmetic and Harmonic Mean</u>

The method of locating a central point between two given points is a distinctly Pythagorean idea.[32] There are many procedures by which to find an average, or mean between two given numbers, which can also be applied to the division of intervallic spaces. The simplest is the arithmetic mean, by which each term in a ratio succeeds the next by the same amount. The description above of stacking a perfect fifth (2:3) and perfect fourth (3:4) to create an octave (1:2) is the prime example of the arithmetic mean.

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29. Ibid.

30. Plato, 20.

31. Crocker, "Pythagorean Mathematics and Music (I)," 193.

32. "Archytas of Tarentum," in *Ancilla to the Pre-Socratic Philosophers*, trans. Kathleen Freeman (Cambridge: Harvard University Press, 1983), 79–80.

When combined to create the ratio 2:3:4, the central term is the arithmetic mean between the two outer terms because each term is greater than the last by 1. Thus the arithmetic mean between an octave is a perfect fourth above the lower member of the octave (see Figure 3).

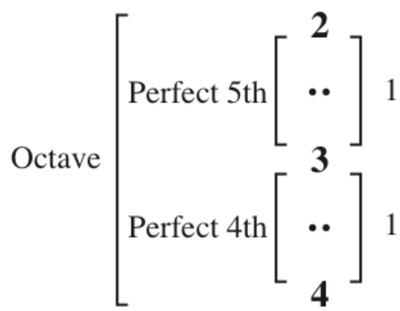

FIGURE 3. An illustration of the arithmetic mean.

This division is not an exact halving function. The lower interval is smaller than the upper. Despite that, both resulting internal intervals are familiar consonances and they create a useful model for the division of the octave, however imperfect.

The practical implication of the arithmetic division was that it was ultimately displeasing to the ear. The unmusical nature of the division is likely the resulting fourth below the fifth.[33] One solution was to flip the order of the arithmetically derived intervals, putting the fourth on top of the fifth. The ratio between the intervals remain the same, aside from the order. At first this was named the subcontrary, and later the

33. Richard L. Crocker, "Pythagorean Mathematics and Music (II)," *The Journal of Aesthetics and Art Criticism* 22, no. 3 (Spring 1964): 328.

harmonic mean, no doubt due to its musical usefulness.[34] The arithmetic and harmonic means can be compared to each other using the ratio 6:12 to represent an octave as seen below (see Figure 4).

The harmonic mean represents the solution to the octave-division problem that proved the most desirable. The perfect fifth below the fourth forms a structure that provides a stable backbone for ancient Greek music and hints at the future developments of Western music.

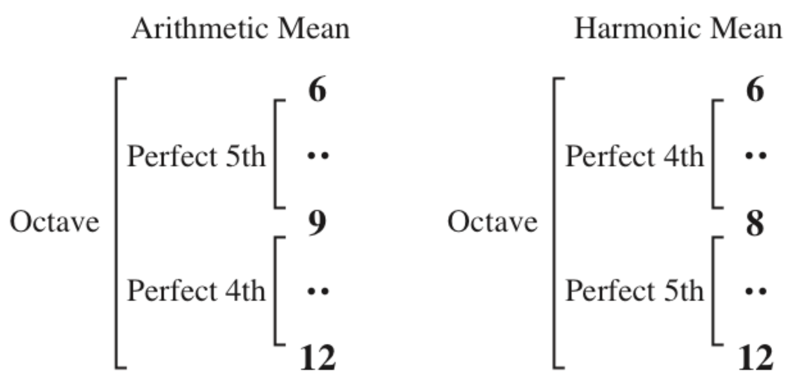

FIGURE 4. An illustration of the arithmetic mean compared to the harmonic mean.

The Geometric Mean

One Pythagorean mean yet remains. The geometric mean, as defined by Archytas, "is when the second is to the third as the first is to the second."[35] In other words, the proportion between the terms of a compound ratio is the same. For example,

34. "Archytas of Tarentum," 79–80.

35. Ibid., 80.

in the ratio 4:6:9, the proportion between the first two terms (4:6) and the following two terms (6:9) is the same (2:3). Therefore, the interval of a major ninth (4:9) has a geometric mean dividing the space equally by a perfect fifth above the lower term, or a perfect fifth below the upper term.

To further illustrate the concept, we can divide the interval of a double octave (1:4). The method of finding the geometric mean between the interval is directly related with the Pythagorean Theorem as detailed above, whereby the mean derives its name. As in the Pythagorean Theorem, the square of the lowest term added to the square of the mean should equal the square of the highest term (see Figure 5). Another, more direct way of calculating the geometric mean between two terms is to take the square root of their product (see Figure 6). Either method results in the compound ratio 1:2:4 where it has the same proportions between each of its sets of terms (1:2).

$1^2 + x^2 = 4^2$; $x = 2$

FIGURE 5. Finding the geometric mean between 1 and 4 using the Pythagorean Theorem.

$\sqrt{(1 \cdot 4)} = x$; $x = 2$

FIGURE 6. Finding the geometric mean between 1 and 4 by the square root of their product.

Comparing Means

A comparison of the three types of means can be made when the double octave is represented by the ratio 10:40 (see Figure 7). The resulting intervals set on arbitrary pitches show that the arithmetic mean creates a larger interval on the top, the harmonic mean creates a larger interval on the bottom, and the geometric mean evenly divides the space.

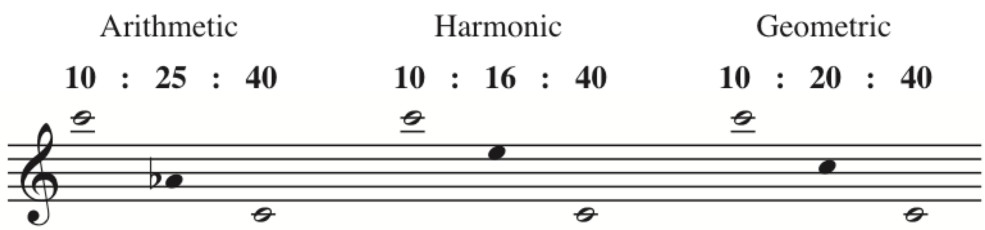

FIGURE 7. A comparison of the three different means.

That the geometric mean most evenly divides the intervals of the octave or compound octave, and that the arithmetic and harmonic means are approximations, is clear. The reason that the early Pythagoreans choose to use the harmonic mean and not the geometric motivates this entire preliminary discussion. As shown above, the space of a double octave divides geometrically, yielding a single octave in the middle. However, when the single octave is split by the geometric mean, we encounter the issue of incommensurability. In order for an octave (1:2) to have the same proportions between each term, the geometric mean must be the square root of two (see Figure 8). As in the

$\sqrt{(1 \cdot 2)} = x$; $x = \sqrt{2}$

FIGURE 8. Finding the geometric mean between 1 and 2 by the square root of their product.

case of Hippasus's right triangle with two sides each equaling one unit where the hypotenuse can be drawn but not described with integers, the interval can be sounded, but it cannot be represented by a whole number ratio. The interval is therefore defined as irrational. For the reason of incommensurability, the resulting interval, and the division of the octave by the geometric mean was abandoned by the Pythagoreans in favor of the harmonic mean.

CHAPTER 3

ANOMALY FOR SOLO FLUTE WITH ELECTRONICS AND ENSEMBLE

General Description

Anomaly is a single-movement chamber work written for solo flute processed with electronics accompanied by a small ensemble. The ensemble consists of oboe, clarinet, horn, trombone, vibraphone, violin, and double bass. The piece is a quasi-concerto or concertino given the prominent role of the flute, an idea supported by an extended flute solo near the beginning of the work and computer processing of the flute's sound exclusively. A MIDI-capable keyboard connected to a computer and sound reinforcement system captures the live sound of the flute and shifts the flute's pitch. In essence, the computer processing as a whole can be thought of as an elaborate polyphonic pitch shifter, capable of producing accurate, microtonal, flute-like timbres with great ease. By using the flute as the sole source of electronically produced sound, the resulting signal processing attempts to augment the sound-creating capabilities of the flute instead of simply adding an extra layer of electronic sound. In the context of the 16-minute piece, the flute and electronics play a complex dependent, yet independent roles that will require further explanation. Before exploring that topic, I will examine the way in which the harmony of the piece develops in relation to the division of the octave.

Division of the Octave

As discussed above, the division of the span of an octave by the Pythagorean cult of ancient Greece inarguably shaped the course of all Western music that followed. Solutions to the puzzle then, in the form of the arithmetic and harmonic mean, led to the prominence of the perfect fourth and perfect fifth. The Pythagoreans dismissed the geometric mean because it produced an interval that could not be expressed by a whole number ratio. In *Anomaly*, I reject the Pythagorean solution and embrace the division of the octave by the geometric mean as the basis for musical composition.

Theory

As explained in Chapter 2, the geometric mean between two lengths of string x and y can be found by taking the square root of their product: $\sqrt{(x \cdot y)}$. In the case of the octave where one string is a length of x, and another is $\frac{1}{2}x$, the resulting geometric mean between the two strings is $x \cdot \sqrt{\frac{1}{2}}$, an irrational string length. In intervallic terms, the geometric mean between an octave yields a tritone. In semitones, the tritone between an octave divides the space evenly with six semitones on either side (see Figure 9).

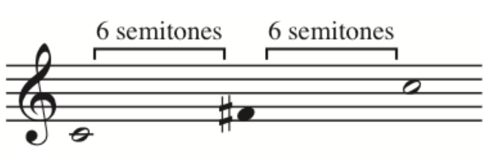

FIGURE 9. The geometric mean between the pitches C4 and C5.

As of yet, the geometric mean has given up only two primary tones with which to work: the root pitch and the tritone away from it. In order to build harmonies from this theoretical basis all of this symmetry and division must be taken a step further: the resulting interval from our first division must itself be divided. Using the geometric mean, the division of a tritone consisting of string length x and $x \cdot \sqrt{½}$ results in a length of $x \cdot \sqrt[4]{½}$. This produces a tone one minor third above the root pitch, centering it between the tritone interval with three semitones on either side (see Figure 10).

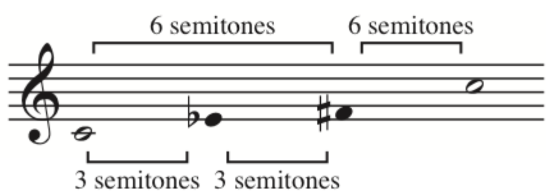

FIGURE 10. The geometric mean between pitches C4 and C5, and between pitches C4 and F♯4.

Should the same operation be conducted on the interval of a minor third, the pitch produced will fall between a major and minor second on an equal tempered scale. As expected, this so-called neutral second[1] will lie equally between the minor third with one and a half semitones above and below it (see Figure 11).[2] This process could, of course,

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1. Jan Haluska, *The Mathematical Theory of Tone Systems* (New York: CRC Press, 2003), xxiii.

2. For notational purposes only, this pitch has been frequency-quantized and rendered as a quarter-tone flat using this symbol: 𝄳.

continue past this first microtonal interval and produce smaller, additional intervals outside of equal temperament.

As the process of halving each successive division continues, a set of intervallic relationships begin to emerge. A pattern from a large outer interval down to incredibly small microtones defines this set of pitches. The opening material of *Anomaly* consists almost entirely of these tones with a root note of A. The resulting pitches from this simple division process are A, B𝄳, C, and E♭. Besides the solo flute, the acoustic instruments are not asked to produce microtones and the B𝄳 is reconciled here in the vibraphone's first measure as both a B♭ and a B (see Figure 12). The choice to "round off" the microtones will be further explained below.

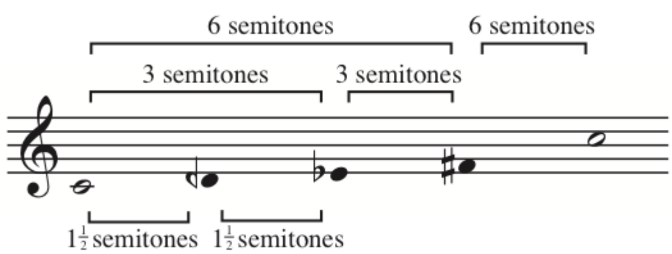

FIGURE 11. The geometric mean between pitches C4 and C5, between pitches C4 and F♯4, and between pitches C4 and E♭4.

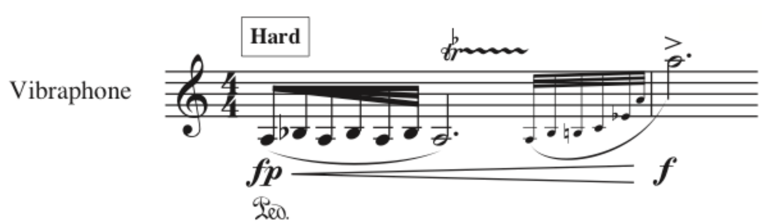

FIGURE 12. Measure 1 of the vibraphone part in *Anomaly*.

Variations in Span

The genesis of *Anomaly* lies in the division of the octave by the geometric mean. The pitches produced by the process of successively dividing that space, however, are eventually limited. In order to create a more harmonically rich and evolving palette, the process must include some variables. One of those variables is the span of the initial, or outer, interval. This is the largest of the subsequent intervals and the space to be divided, from which all following intervals originate. Thus far, the outer interval has been the space of one octave. This does not remain the case for much of the piece. In fact, the true outer interval of the opening material as discussed above is not an octave, but rather a double octave. The double octave, when divided by the geometric mean, yields a single octave. What follows from that interval has already been detailed above.

The double octave is the largest space divided in *Anomaly*. Intervals like an octave plus a major seventh, or an octave plus a minor seventh, all the way down to the single octave, form the great majority of the harmonic language. Every outer interval produces its own subset of inner divisions, each with its own character. The division of the outer interval of an octave and a minor sixth, for example, produces a minor seventh, followed by a perfect fourth, followed by a subminor third[3] (see Figure 13).[4] Thus, by varying the size of the outer interval, a wide array of harmonic complexity can be created.

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3. Hermann von Helmholtz, *On the Sensations of Tone* (London: Longman, 1895), 195.

4. For notational purposes only, this pitch has been frequency-quantized and rendered as a quarter-tone sharp using this symbol: ‡.

FIGURE 13. The resulting pitches from dividing the space of an octave plus a minor sixth by the geometric mean.

Variations in Direction

So far, a general shape has emerged in terms of interval size. Starting from the largest, outer interval, the resulting divisions in relation to the root pitch produce ever-decreasing intervals. To say it another way, the interval size expands the further it gets away from the root pitch and contracts the closer it gets to the root pitch. Another variation from the basic division of the octave is to invert this shape by changing the direction of the division. Instead of dividing the outer interval downward towards a root pitch, the divisions are made upward toward the higher member of the outer interval. In doing so, the resulting intervals from an outer interval of a double octave would be an octave, an augmented eleventh, followed by a major thirteenth, followed by a neutral fourteenth (see Figure 14).[5] Since these are the inverse intervals of the basic division of the octave as detailed above, the harmonies will be related to those described earlier. To further clarify the shape of this alternate direction, they can be referred to as inwardly-expanding, outwardly-contracting divisions, meaning that interval size quickly expands away from the root pitch and subsequently shrinks as it approaches the outer note.

5. Haluska, xxiii.

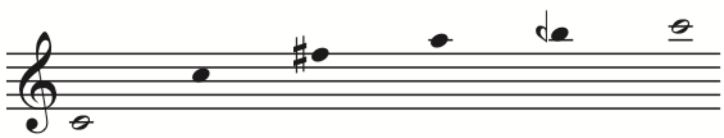

FIGURE 14. The resulting pitches from dividing a double octave upward by the geometric mean.

Variations in Division

The variations in outer interval size and division direction create a fairly large and diverse harmonic palette. Coupled with changes in the root pitch, there is a seemingly endless combination of pitches and intervals. However, in the experiments conducted dividing the octave, the possibility of other potential divisions could not go unexplored.

Dividing the octave into three parts by the geometric mean results in reducing the outer interval by a third at every division. The resulting interval is further divided into three parts and reduced by a third. So, for example, to divide a double octave by thirds results in a major tenth, followed by a neutral seventh, followed by a perfect fifth, followed by a series of ever-diminishing microtonal intervals (see Figure 15).

FIGURE 15. The resulting pitches from dividing the double octave downward in thirds by the geometric mean.

Variations in outer interval size and division direction can be applied to this type of division as well. Compared to the harmonic language of half divisions, the harmonies of third divisions sound different, yet related because of the geometric process. The inner pitches of half divisions (D𝄳, E♭, F♯) move dramatically upward in third divisions (G, B𝄳, E), expanding the lower intervals and contracting the upper intervals.

Geometric divisions can continue past half divisions, and third divisions, infinitely separating the outer interval into smaller and smaller pieces. In *Anomaly*, I have chosen to continue the division process up through eighth divisions. Appendix A contains an exhaustive visual explanation of each series of divisions, including fine detail of the microtonal fractions of a tone generated by this process. As the divisions increase, there is a natural, step-wise progression away from the original sound world.

The formal structure of *Anomaly* is based on a series of increasing division variations. As stated previously, the opening material comes from the basic half division of the span of the double octave. As the piece progresses, the division type moves from half divisions, to third divisions, continuing through the subsequent possibilities before ending on eighth divisions. Each stage in the progression of these divisions tends to mark a significant moment or a sectional demarcation in the piece. An outline of these divisions with measure numbers is shown in Figure 16. Given the multiplicity of variation and the resulting harmonic complexity, the necessity of the computer processing becomes evident. It is that subject I will address next.

Type of Division

1/2	1/2	1/3	1/4	1/5	1/6	1/7	1/8
m. 1	cadenza	m. 26	m. 51	m. 74	m. 128	m. 207	m. 251

Measure Number

FIGURE 16. The formal structure of *Anomaly* based on the intervallic division type.

<u>Function of the Processing</u>

In *Anomaly* the electronic computer processing has a multifaceted role. On the one hand, the processing is of practical necessity. Many of the harmonies produced by the division process are precise microtones, beyond the reasonable demands made of most performers. The computer plays a role that it alone can fulfill by reproducing accurate microtones.

The processing also plays a role as an augmentation of the flute. Soon to be explained below, the sole source of electronic sound comes from a microphone attached to the flute. The resulting processing sounds like a flute. Even the dynamic and articulative inflections produced by the flute player will translate into the processed sound in real time. Therefore, the processing in one sense can be though of as "flute plus."

The computer processing is not tethered to the flute part completely. While the flute initiates all computer sounds, the flute source can be virtually frozen in time and stored to be played back independently. All division processes can be applied to and triggered by a frozen sound just as if the flute were then playing that sound into the microphone. In this way, the role of the processing achieves a complex flute-dependent freedom that allows it to have its own voice in addition to augmenting the flute.

Finally, the goal of the computer processing is to blend naturally with the ensemble. In having an independent voice and creating flute-like timbres, the processing sounds and acts like an acoustic instrument. The significant difference, however, is its ability to accomplish polyphonic microtonal pitch shifting. In order fully to understand the processing, it is necessary to turn to the underlying programming, built in Cycling 74's Max/MSP.

Description of the Max Patch

I realized the computer processing in *Anomaly* though the visual programing environment of Cycling 74's Max/MSP. I built the patch in interconnecting modules that can be generally represented in a block diagram (see Figure 17).

The patch has two inputs: an audio input to capture the flute's sound via an attached microphone, and a MIDI data input to accept keyboard messages from a MIDI-capable keyboard. The audio from the flute first passes through a pitch detector module that analyzes the fundamental frequency of the incoming sound and sends that message to the division multipliers. The audio continues through to the freezer module, which provides the ability either to let the audio pass through in real time, or to capture the spectral characteristics of one moment of sound and replay them, virtually freezing the sound. The composer and programmer Jean-François Charles developed this method of freezing an audio signal using Jitter matrices to store FFT information.[6] The flute audio,

6. Jean-François Charles, "A Tutorial on Spectral Sound Processing Using Max/MSP and Jitter," *Computer Music Journal* 32, no. 3 (Fall 2008): 100.

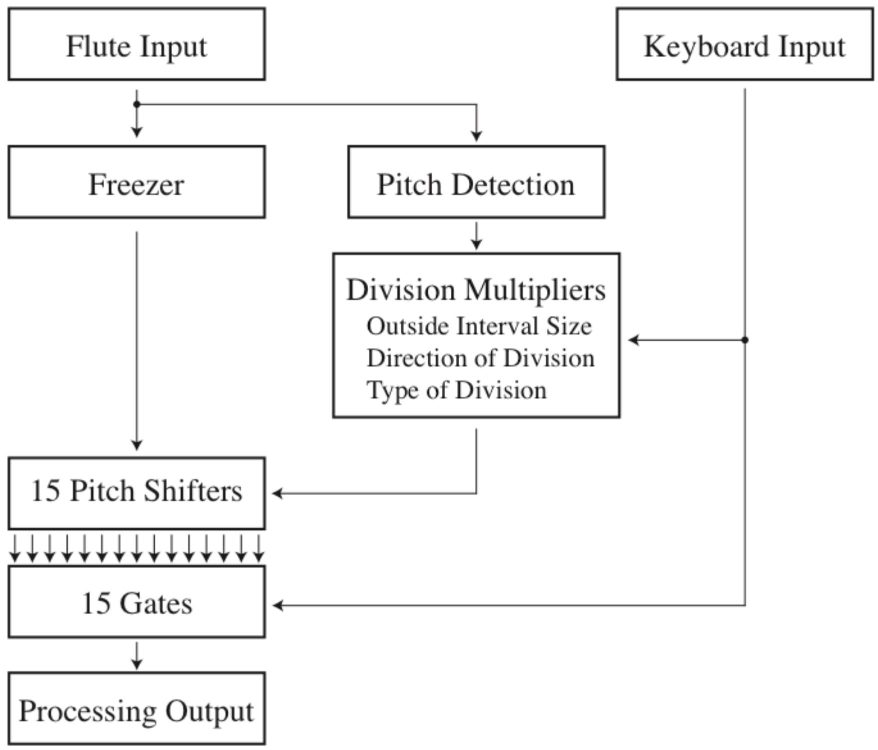

FIGURE 17. A block diagram of the computer processing used in *Anomaly*.

frozen or real-time, passes to fifteen individual pitch shifter modules. Each of these pitch shifters is capable of taking the audio and changing its pitch without affecting playback speed. The pitch shifters are also independent from each other, allowing fifteen different degrees of pitch shifting.

In order to coordinate the precise, polyphonic pitch shifting that is required for *Anomaly*, a fairly complex mathematical equation takes the fundamental frequency message from the pitch detector, in tandem with MIDI keyboard messages, and produces fifteen different degrees of pitch shifting. The three principal variables involved in the equation accomplish what has already been discussed in some detail above. These three variables account for the outer interval size, the direction of the division of that outer interval, and the division type. They fit into a standard equation used to calculate a frequency a specific distance of semitones away from a given pitch (see Figure 18).

$$\text{New Frequency} = (\text{Center Pitch}) \times (\text{Outside Interval})^{\wedge}\left(\frac{1}{(\text{Type of Division})^{\wedge(\text{Which Division})}}\right)$$

FIGURE 18. The mathematical equation used to account for each of the variables used in *Anomaly*.

After passing through the pitch shifter, fifteen specific pitch shifted versions of the original audio are generated. These are then gated and individually triggered when a corresponding MIDI note is received. The patch can then output any of these 15 new pitches individually, or in any combination.

Description of the Keyboard Layout

Any standard MIDI keyboard can be used in conjunction with Max/MSP in this piece. The primary function of the keyboard is to augment the expressive capabilities of the flutist by triggering particular pitch shifted versions of the flute's sound in real time.

The keyboard layout depicted in Figure 19 shows every functioning key. The lower end of the keyboard has mode switches that change the direction of divisions (outwardly expanding, or outwardly contracting), the size of the outer interval, and the type of division (half divisions, third divisions, fourth divisions, etc). The upper end of the keyboard has the individual division triggers. Key C5 will reproduce the given pitch unaltered, while the keys on either side will shift the pitch up or down.

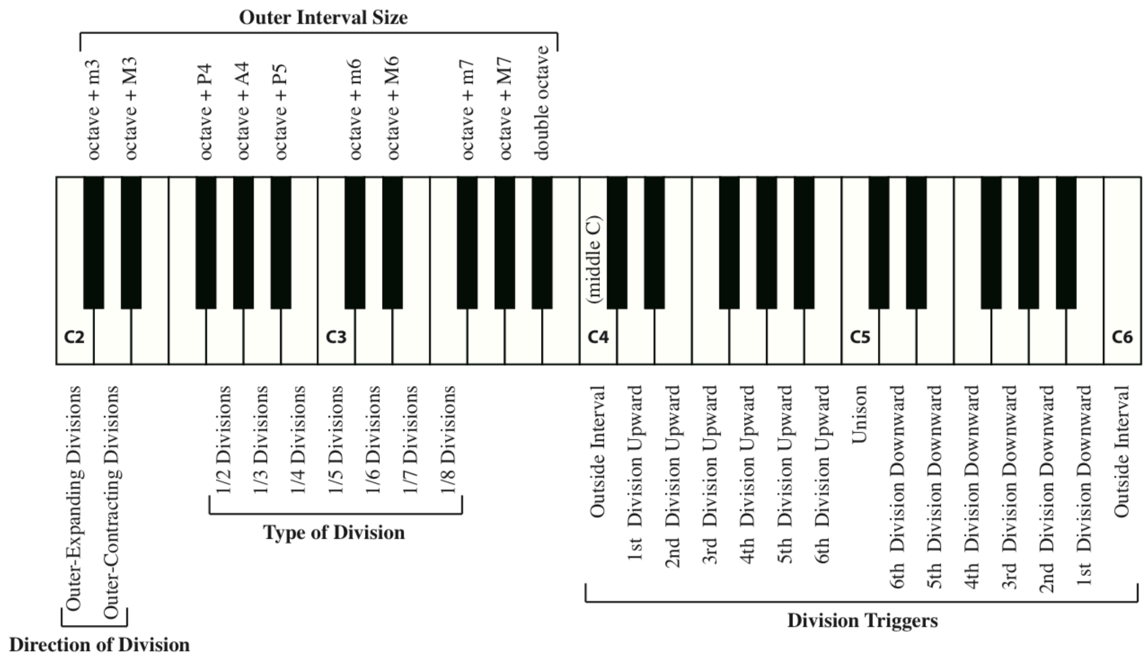

FIGURE 19. A diagram of the keyboard's layout by function.

The diagram below (see Figure 20) gives an example of the resulting intervals when keys C2, G2, and B♭3 have been pressed. Note that these keys do not need to remain depressed and do not produce any sound by themselves since they are functioning only as switches for the patch.

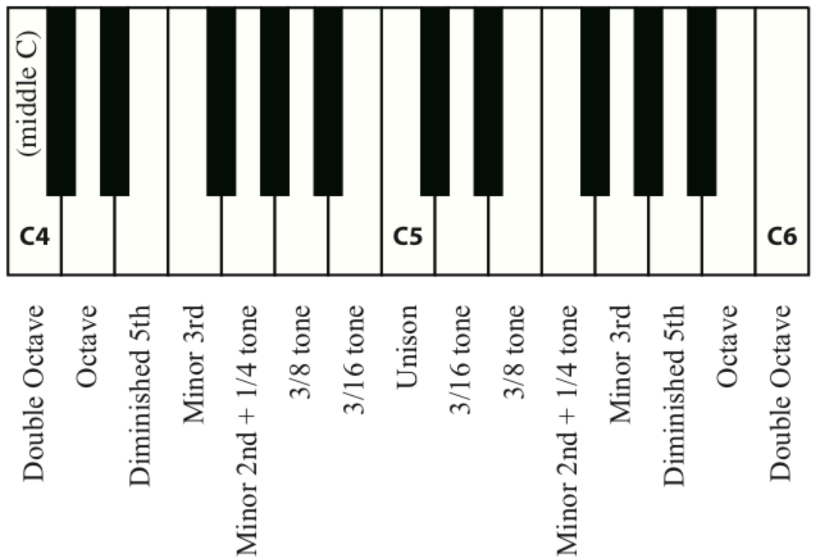

FIGURE 20. The resulting intervals when the keys C2, G2, and B♭3 are pressed.

The staff below (see Figure 21) shows the 15 possible resulting pitches if the flute plays C4. Each of these pitches can be triggered one at a time, in multiples or all at once.

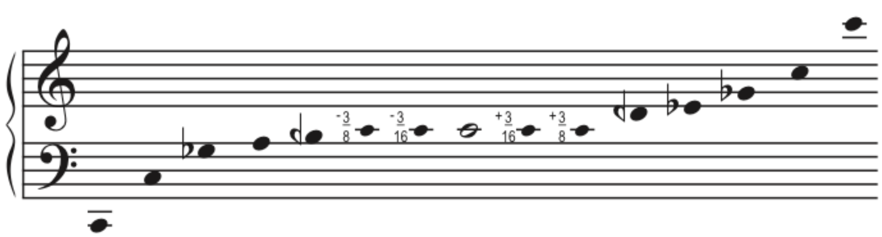

FIGURE 21. The resulting pitches when the keys C2, G2, and B♭3 are pressed and the flute is playing a C4.

The sustain pedal is responsible for switching between real-time processing and freezing a moment of sound in time and processing it for triggering later. Pedal markings in the piece must be strictly observed as they are crucial to capturing and reproducing specific pitches. In addition, the keyboard player must pay careful attention to hand-foot coordination as *Anomaly* employs the pedal using non-traditional technique. Finally, the expression pedal is used to attenuate the master output from the computer. This adds an additional level of dynamic expression to the computer processing. In short, *Anomaly* requires a highly skilled keyboardist who plays a central role as a performer in the piece.

Realization of Microtones

Anomaly makes limited use of microtones outside of the computer processing. The nature of the process driving the harmonic language demands the type of microtonal accuracy that is quite possibly beyond the reasonable capability of most performers. Given this dilemma, I had to consider what would produce the most effective solution. In the case of *Anomaly*, the ensemble is spared from the rigor of producing these microtones. Instead, the score uses the closest approximate equal tempered pitch.

The score substitutes a careful selection of frequency-quantized pitches in place of microtones. Microtones closer to the upper end of a pitch are rounded upward, while microtones closer to the lower end of a pitch are rounded downward. In the case of exact quarter-tones, both pitches surrounding the quarter-tone are used. For example, given a D4 a third-tone sharp, the pitch will be quantized down to an equal-tempered D4. If, on the other hand, the D4 is two-thirds of a tone sharp, the quantized equal-tempered pitch will be moved up to an E♭4. In the special case that the D4 is exactly a quarter-tone sharp, the pitch will be split into an equal-tempered D4 and E♭4 (see Figure 22).

FIGURE 22. Three microtones are frequency-quantized to equal-tempered pitches.

In this way, the pitch material in *Anomaly* is approximately built from the precise, pitch-shifting calculations done in the computer processing. Due to this disparity between precision and approximation, the piece embodies a sort of tension that could easily become overwhelmed in dissonance. However, special attention has been given to the orchestration of the piece to avoid direct conflict between a precise pitch and its approximate equivalent. The resulting sonic world created by this geometric division process and its computer-assisted realization creates something that is, in a way, an extension of harmonic reality itself.

CHAPTER 4

ANOMALY AS AN EXTENSION OF SPECTRALISM

Now that I have presented the theoretical background and practical implementation of *Anomaly*, I will argue that the piece is closely aligned with some of the tenets held by composers associated with the spectral movement. After a brief overview of spectralism, I will compare my composing philosophy with that of the so-called spectralists.

Spectralism

Before relating *Anomaly* to spectralism, a brief historical survey of the movement is required. Due to the relatively recent emergence of spectralism, the philosophies shared by its proponents are still evolving. One hallmark trait of the movement is a fascination with perception, which typically manifests itself in an exploration of the acoustic properties of sound and the malleability of time.[1] The interest in sound's inherent properties is nothing new, as other critics and music historians have pointed out, "The attempt to relate musico-cultural activity to (supposedly) natural laws of acoustics has been a mainstay of musical theory since the time of the ancient Greeks."[2] However, a key difference in the birth of spectralism was the availability of technology that could analyze and resynthesize a sound. In fact, the link between technological development

1. Julian Anderson, "A Provisional History of Spectral Music," *Contemporary Music Review* 19, no. 2 (2000): 7–8.

2. Ibid., 8.

and musical aesthetics was so closely linked at this time, that a new term, *compositeur en recherche*, describing a composer-as-scientist, was proposed to account for the growing reliance on computer-aided composition.[3]

The exploration of the acoustic properties of sound by spectral composers such as Gérard Grisey and Tristan Murail in the mid-1970s relied on the development of the spectrogram, which, by using the Fast Fourier Transform function, or FFT, could break down a complex sound into its individual sinusoidal components.[4] In this way, a sound's spectrum can be described in terms of a fundamental and overtones, regardless of the complexity of the sound. Grisey and Murail used the relationships in the overtone series as their foundational pitch material, taking the musical property of timbre and making it the focus of their compositions.[5] Composers preceding spectralism, such as Stockhausen, Scelsi, Messiaen, and Varèse, all experimented with the centrality of timbre,[6] but Grisey was arguably the first to blur the lines between timbre and harmony in his monumental work *Les éspaces acoustiques*.[7] Eleven years in the making and composed of six movements that can be played together or separately, Grisey built *Les éspaces*

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3. Eric Daubresse and Gérard Assayag, "Technology and Creation—The Creative Evolution," trans. Joshua Fineberg, *Contemporary Music Review* 19, no. 2 (2000): 64.

4. Jont B. Allen and Lawrence R. Rabiner, "A Unified Approach to Short-Time Fourier Analysis and Synthesis," *Proceedings of the IEEE* 65, no. 11 (November 1977): 1558.

5. François Rose, "Introduction to the Pitch Organization of French Spectral Music," *Perspectives of New Music* 34, no. 2 (Summer 1996): 6–7.

6. Anderson, "Provisional History," 8–14.

7. Julian Anderson, "Gérard Grisey," in *The New Grove Dictionary of Music and Musicians*, 2nd ed. (London: Macmillan, 2001), 10:428–9.

acoustiques upon the central idea of exploring acoustical timbres by orchestrating their spectral components. *Les éspaces acoustiques* is one of the principal works of spectralism and makes for a suitable comparison to *Anomaly* in terms of its aesthetic point of view.

Working in Overtones

At this point, I will give careful consideration to the terms "harmonic" and "overtone." Harmonics should be understood to be frequencies above a fundamental that are related to the fundamental in simple whole number ratios.[8] Given a fundamental frequency of 100 Hz, the frequencies 200 Hz, 300 Hz, and 400 Hz would all be considered harmonics. Overtones, on the other hand, have the much broader definition of any frequency component of a sound above the fundamental.[9] Overtones take into account the idea of inharmonicity, that is, intervallic ratios that cannot be expressed by whole number ratios.[10] Under this broader definition, intervals created by irrational ratios as described earlier and used in *Anomaly*, can be thought of as overtones.

Harmonics

One of the fundamental tenets of spectralism is an adherence to the acoustic properties of sound. The opening material of *Partiels*, one of the central movements of *Les éspaces acoustiques*, is often used as the prime example of this, being based on the

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8. Guy Oldham, Murray Campbell, and Clive Greated, "Harmonics," in *The New Grove Dictionary of Music and Musicians*, 2nd ed. (London: Macmillan, 2001), 10:854.

9. Murray Campbell, "Overtones," in *The New Grove Dictionary of Music and Musicians*, 2nd ed. (London: Macmillan, 2001), 18:821.

10. Rose, 9.

spectral properties of a trombone's lowest E.[11] Harmonic components present in the sound of the fundamental E are written into the parts of the ensemble. Figure 23 shows Grisey's orchestration of the trombone's timbre. The natural phenomena of the harmonic series becomes the genesis of an entire sound world.

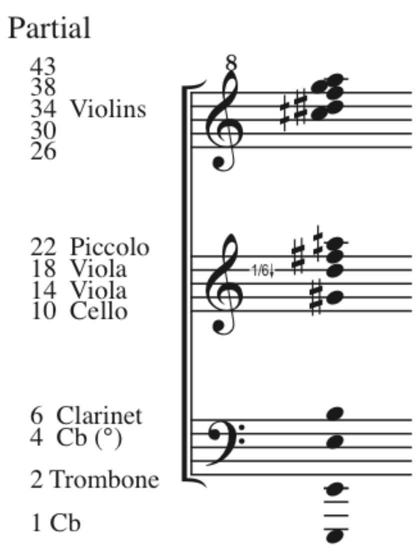

FIGURE 23. Grisey's orchestration of the trombone's spectral components in *Partiels*.

Working with a given fundamental and generating pitches upward is not Grisey's only method of deriving material for his piece. The span of the frequency range can be staggering given the large intervals close to the fundamental. Unless the fundamental is significantly low, the tessitura in which many of the closely spaced upper harmonics occupy will be stratospheric. For this reason, Grisey developed techniques that generate material which exhibits a great deal of harmonicity without technically being harmonic.

11. Ibid., 8–9.

In other words, while the central concept of spectralism in *Les éspaces acoustiques* lies in its adherence to the harmonic series, much of its actual content is better expressed as overtones.

<u>Overtones</u>

Many of the techniques used in creating inharmonic overtones find their origins in electronic procedures that have been adapted for acoustic instruments.[12] This is not surprising given the link between the development of spectralism and the emergence of computer-aided acoustic analysis. Some examples of electronically derived procedures that have been adapted for acoustic composition include ring modulation and frequency modulation.

Another process used by Grisey involves creating virtual fundamentals for a given set of sounds. Because the harmonic series continues upward in ever-smaller intervals, any group of pitches can be related to one another by a fundamental frequency, which is sometimes a very low pitch.[13] This fundamental is called virtual because it is arguable whether or not it can be perceived by a listener, and it is often unable to be sounded by an instrument in the correct octave. Composers have used this methodology to generate material that is spectral and related to the harmonic series for their pieces, even though the relation is one that is artificially constructed and difficult, if not impossible, to hear.

The concept of a virtual fundamental is not critically flawed, however. While the typical measure used to judge perceptibility is the natural harmonic series, there is no

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12. Ibid., 11.

13. Joshua Fineberg, "Guide to the Basic Concepts and Techniques of Spectral Music," *Contemporary Music Review* 19, no. 2 (2000): 98.

guarantee that this is the only spectrum for which the ear has an affinity. An artificial spectrum, one not naturally occurring but generated by a computer algorithm or a composer's imagination, allows for any sort of inharmonic relationship.[14] These relationships are instead dependent on our definition of overtones and, again, often result from electronically derived procedures. One example of an artificial spectrum is the subharmonic series wherein the intervallic relationships of the harmonic series are inverted and overtones are created below a given fundamental.[15]

It is fitting, then, to call the process involved in *Anomaly* one that is capable of generating types of artificial spectra. A given pitch to be processed can be understood in some sense to be a fundamental, upon which the upward divisions are built. These resulting pitches are overtones in that their frequencies are related through the geometric mean to the fundamental. The inverse process of generating pitches downward is analogous to the artificial spectrum of the subharmonic series. Both operations simply take a series and mirror the generated intervals above or below the fundamental. The central point in aligning *Anomaly* with spectral thinking is the shared application of overtones as produced by an electronically derived process.

Working in Extended Time

Dissecting and reconstructing the spectral elements of a sound is only one technique used by spectral composers. As Grisey said in a 1996 interview, "The departure point of spectralism was . . . the fascination for extended time and for continuity . . . . That is really the starting point of spectralism and not the writing of

14. Ibid., 92–3.

15. Rose, 15–6.

spectrums or whatever."[16] In Grisey's assessment, the true focus of spectralism lies in the exploration of time as something that could be dilated and stretched. In fact, in his 1980 lecture at Darmstadt entitled "*Tempus ex Machina*: A Composer's Reflections on Musical Time," Grisey elaborates on an embodied time, composed of a skeleton, flesh, and skin.[17] Grisey's fascination with time and the methods he employed have some bearing on time elements in *Anomaly*.

Proportional Time

Aside from a brief introduction, the entire opening half of *Anomaly* takes place in either unmetered or proportional time. The extended flute and electronic solo section lacks any concrete tempo indication. In some ways very similar to a cadenza, the flute and electronics have a rhythmic free will. The notation is spaced in such a way to indicate quicker rhythmic activity when note heads are closer together and slower activity when further apart. This notation is similar to Grisey's notation for his *Prologue for Solo Viola*, the opening movement of *Les éspaces acoustiques*. Beamed notes indicate phrase markings and breath markings indicate shorter and longer pauses between phrases.

A type of measured proportional notation is implemented in *Anomaly* when the ensemble enters on the downbeat of measure 26. Barlines represent passing seconds and are bounded by darker barlines into groups of seconds (see Figure 24).

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16. David Bundler, "Interview with Gérard Grisey," Musical Time Articles, Interviews, and Essays, http://www.angelfire.com/music2/davidbundler/grisey.html (accessed January 28, 2012).

17. Gérald Grisey, "*Tempus ex Machina*: A Composer's Reflections on Musical Time," trans. S. Welbourn, *Contemporary Music Review* 2 (1987): 241.

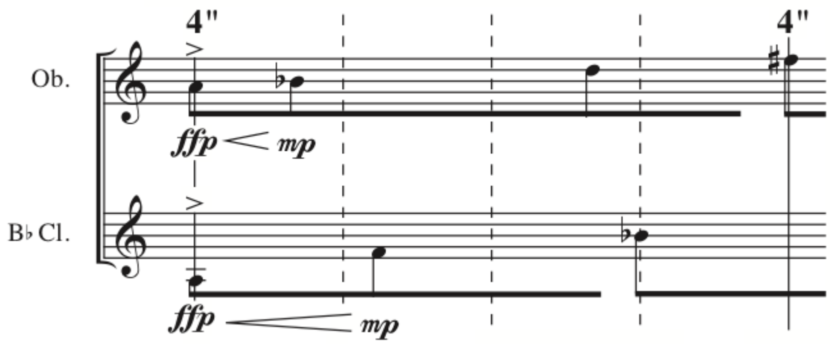


FIGURE 24. Measure 26 from *Anomaly* showing an example of the proportional notation.

The performer is given pitches whose durations are relative to their horizontal placement in the measure. In much the same way, Grisey uses familiar time signatures with proportional notation in *Partiels*. In this way, the rhythmic gestures are freed from traditional measured constraints.

The use of unmetered and proportional time in *Anomaly* is to present the listener music that, "treats [time] as a constituent element of sound itself."[18] The collective result is a sonic mass that expands, contracts, and moves nebulously within the chronological time frame. Time seems to stand still, go backward, and suddenly rush forward.

Strict Time

The technique of strict, or traditionally metered time, acts as bookends in *Anomaly*, surrounding a large section of unmetered time. While seemingly ordinary in function, the metered notation serves two distinctly different functions in relation to the proportional time. First, the strict time acts as a foil to the unmetered time. Where the

18. Gérald Grisey, "Did You Say Spectral?," trans. Joshua Fineberg, *Contemporary Music Review* 19, no. 3 (2000): 1.

proportional notation allows some rhythmic ambiguity and room for performer interpretation, the metered time must be exactingly precise. The section beginning at measure 74 demands a rigorous adherence to the newly introduced metered notation (see Figure 25). The strict time here helps provide a contrast to the previous section of

FIGURE 25. Measures 88–89 in *Anomaly* showing an example of the strict time.

proportional time. Secondly, the sections of strict time near the end of the piece imitate the gestures of the earlier proportional time. In this way, the *Anomaly* makes certain rhythmic connections between two separate sections of the piece that use two different approaches to time. The result is something other than direct imitation or variation; the gesture remains the same, but the context changes. Using two methods of notating similar rhythmic material indicates a development of time as progressively changing over the course of the entire work.

Progressive Time

On a macro level, *Anomaly* exhibits a distinctly spectral view of time as something that can be expanded and contracted. A sense of time as a progression develops from the beginning of the piece, starting from the unmetered stillness of the flute and electronic solo. Rhythmic activity in the flute increases in complexity before the ensemble joins in a chronometrically based proportional sound collage (see Figure 26). Around measure 38, the amorphous texture produced by the ensemble begins to clarify in entrance and gesture. The punctuated attacks at measure 51 point towards a more organized sense of rhythm while continuing to obscure a real pulse (see Figure 27). Finally, the accelerando into measure 74 intensifies the arrival of a solid metrical grounding. The remainder of the piece can be heard as a continual push from strict time into a sense of time that rushes wildly forward, sounding less and less under tight control. The progress of time in the piece in sum is a dynamic excitation, building up from stillness, to control, to the edge of control, before approaching stillness once again.

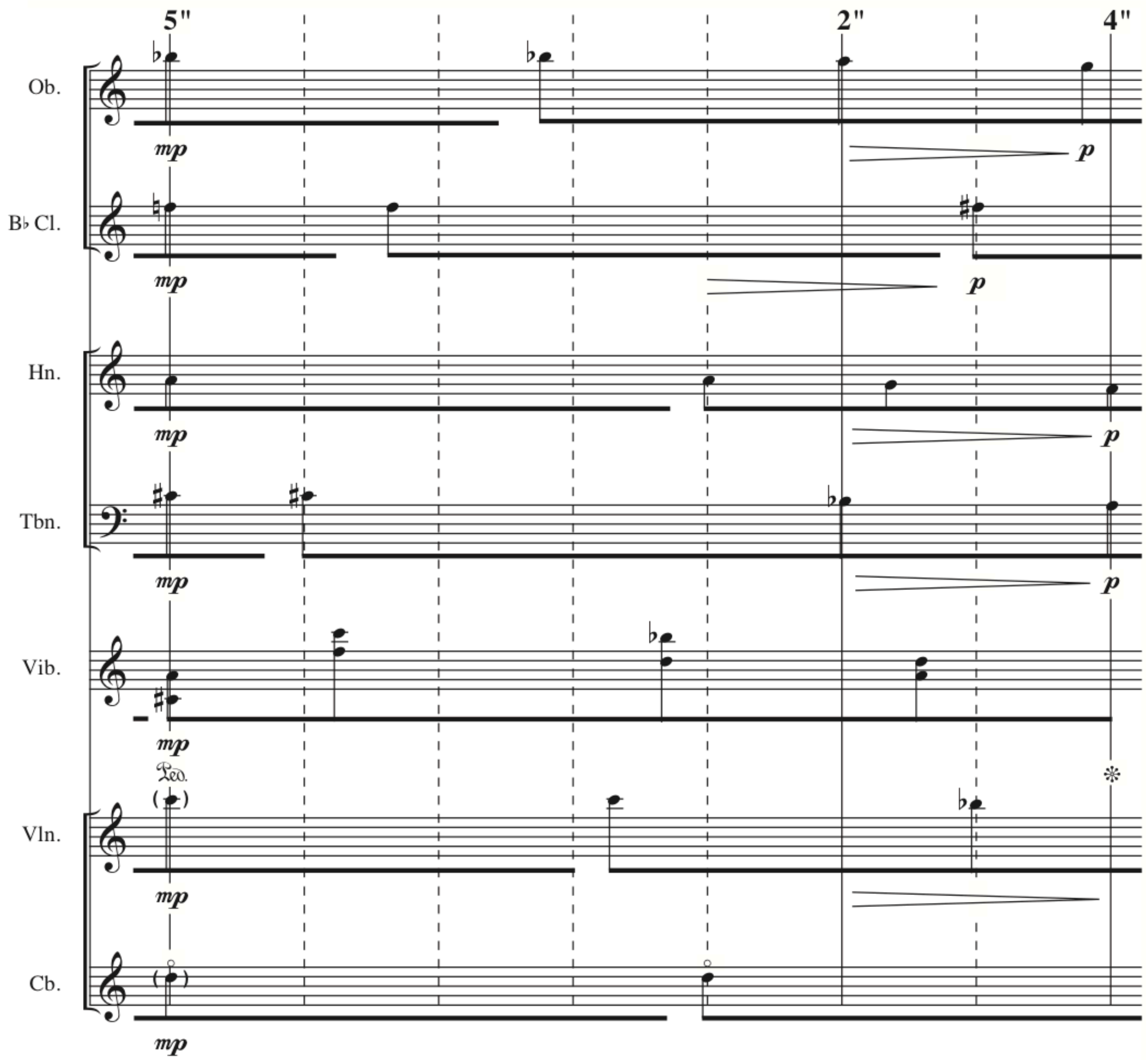


FIGURE 26. Measures 30–31 in *Anomaly* showing how proportional time assists in creating a nebulous sound mass.

FIGURE 27. Measures 53–54 in *Anomaly* showing a progressive development of rhythmic precision.

CHAPTER 5

CONCLUSION

The ancient paradigm of the Pythagorean division of the octave by the harmonic mean established the way for future musical developments. At first, it emboldened early music theorists to proclaim the perfection of the fourth and fifth. It helped segment the octave into equal parts resulting in scalar systems such as the Greater Perfect System and modal church systems. Later, it gave rise to tonic-dominant relationships and tonality.

The effects of the harmonic division are far-reaching, but in the end, they are just one paradigm, reinforced by many composers and music theorists over centuries. Referring to change within scientific paradigms, philosopher Thomas Kuhn said, "The more precise and far-reaching [a] paradigm is, the more sensitive an indicator it provides of anomaly and hence of an occasion for paradigm change."[1] The twentieth century has proven to be such an occasion for large-scale change in musical paradigms. Viewing all musical elements through a spectral lens is one example of a relatively recent shift in paradigm.

The purpose of presenting the subjects in this paper in relation to *Anomaly* is to highlight the anomalous nature of the geometric mean and show that its theoretical implications are musically viable. The octave lies in the vast continuum of frequency space, and just as Pythagoras and his followers explored the nature of sound itself, we too

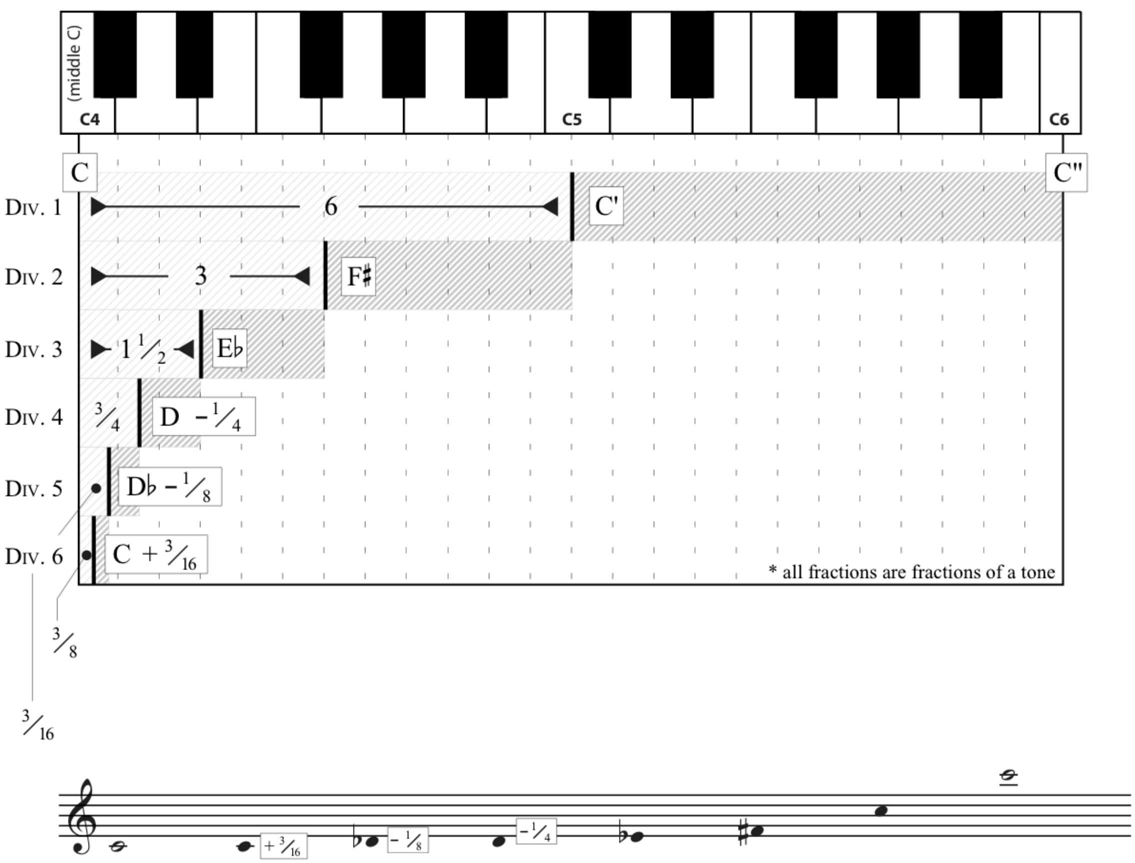
1. Kuhn, 65.

are able to mine this space for new divisions. *Anomaly* shows that incommensurability of the geometric division of the octave, while rejected for the large part of Western music, is ripe with new musical possibilities.

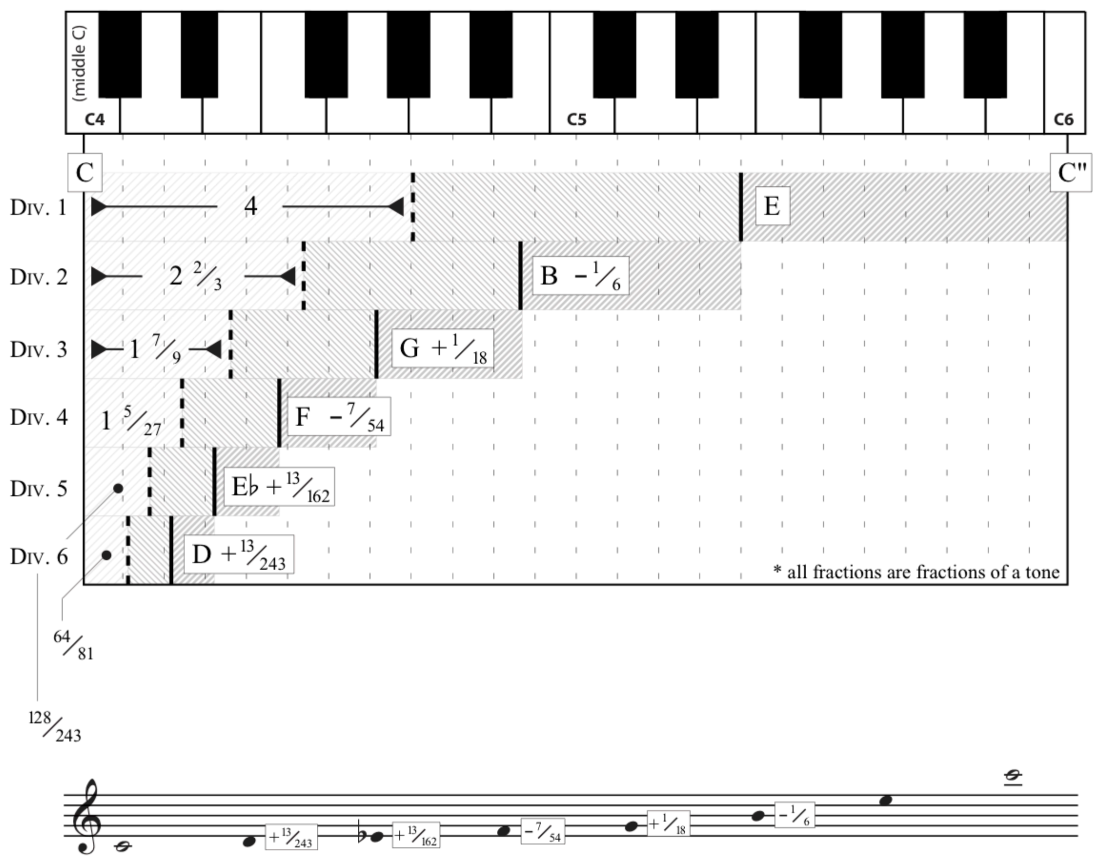
APPENDIX A

ILLUSTRATED DESCRIPTION OF DIVISION TYPES

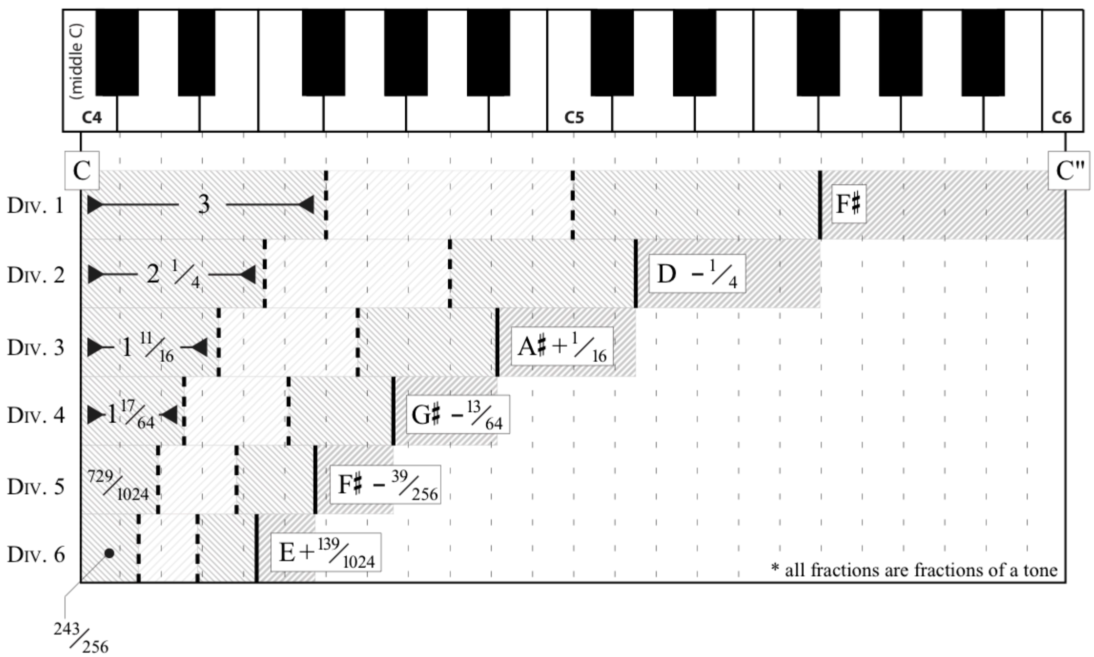
Half Divisions



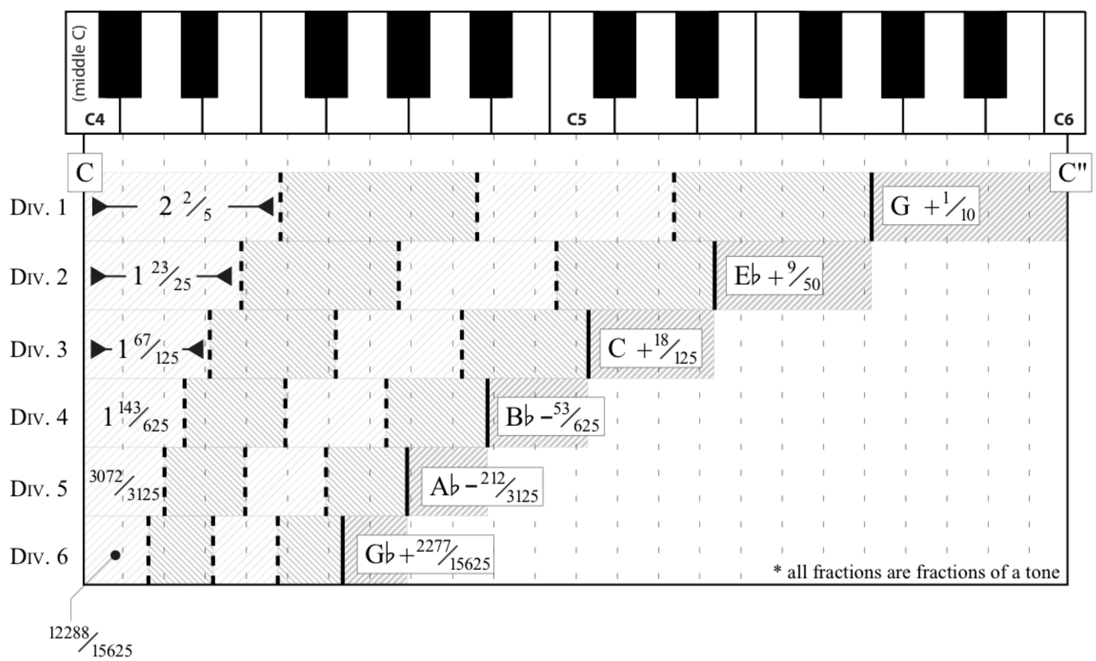
Third Divisions

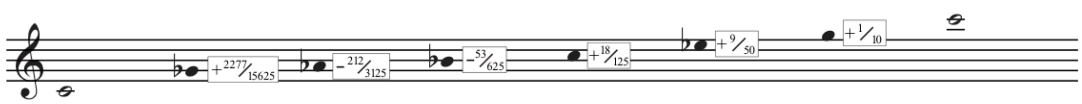


Fourth Divisions

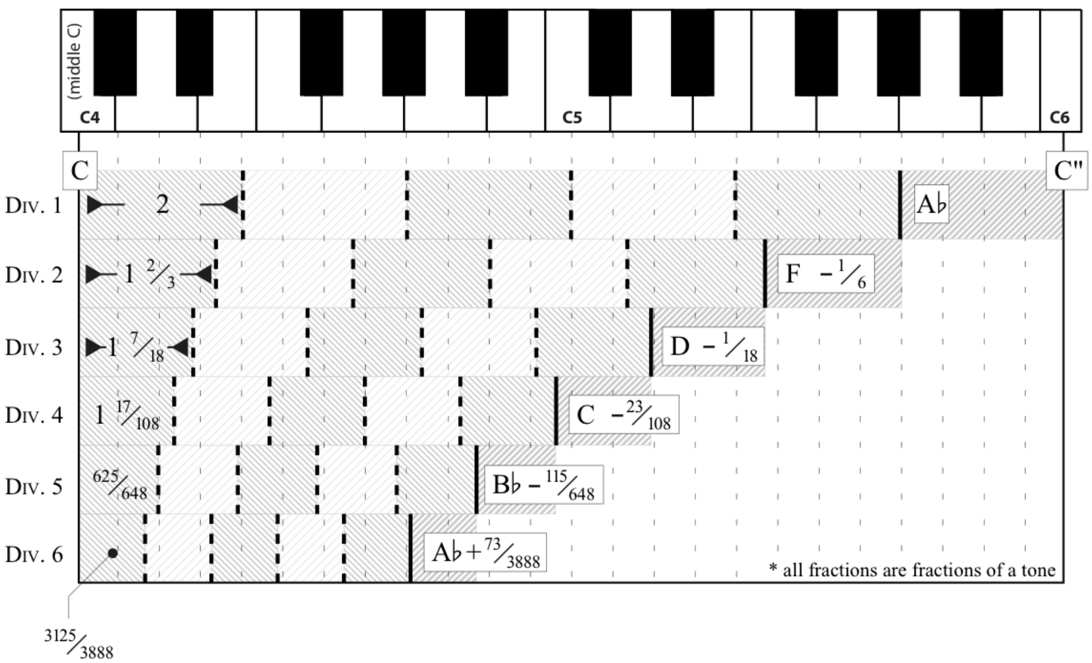


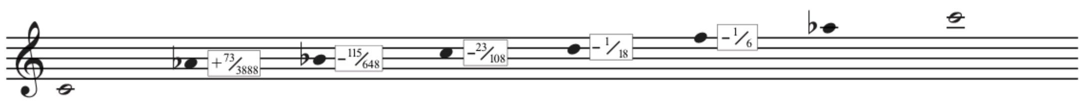
Fifth Divisions



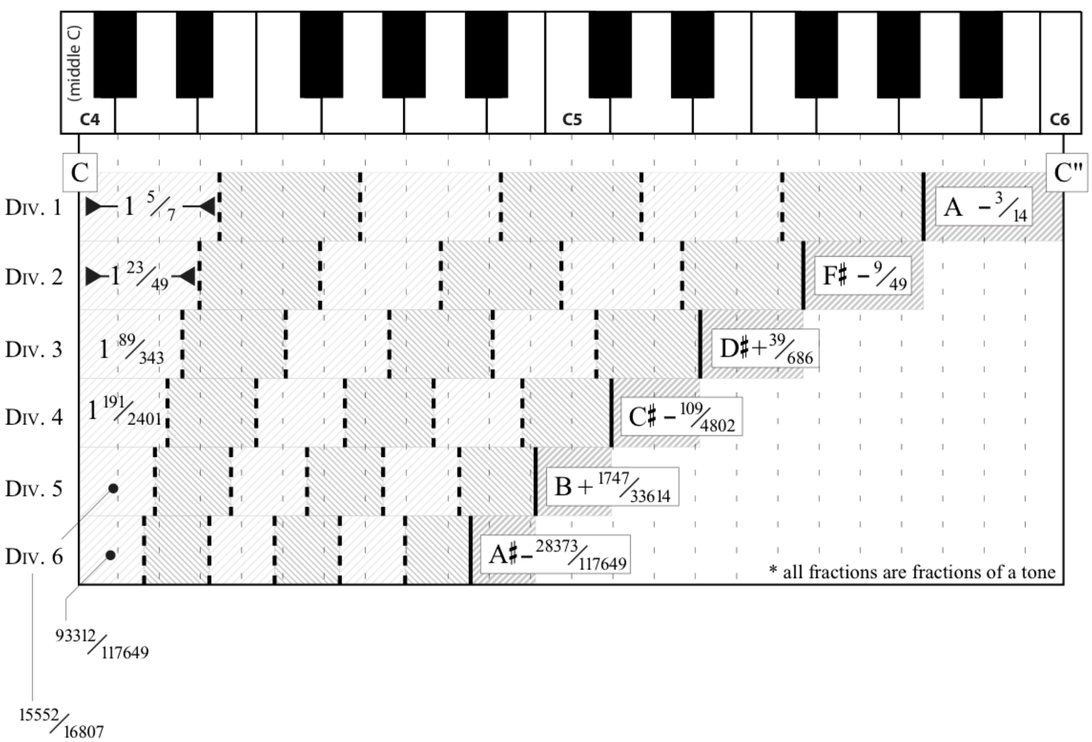


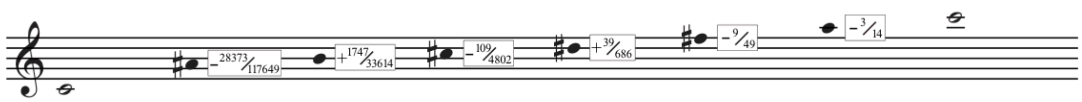
Sixth Divisions



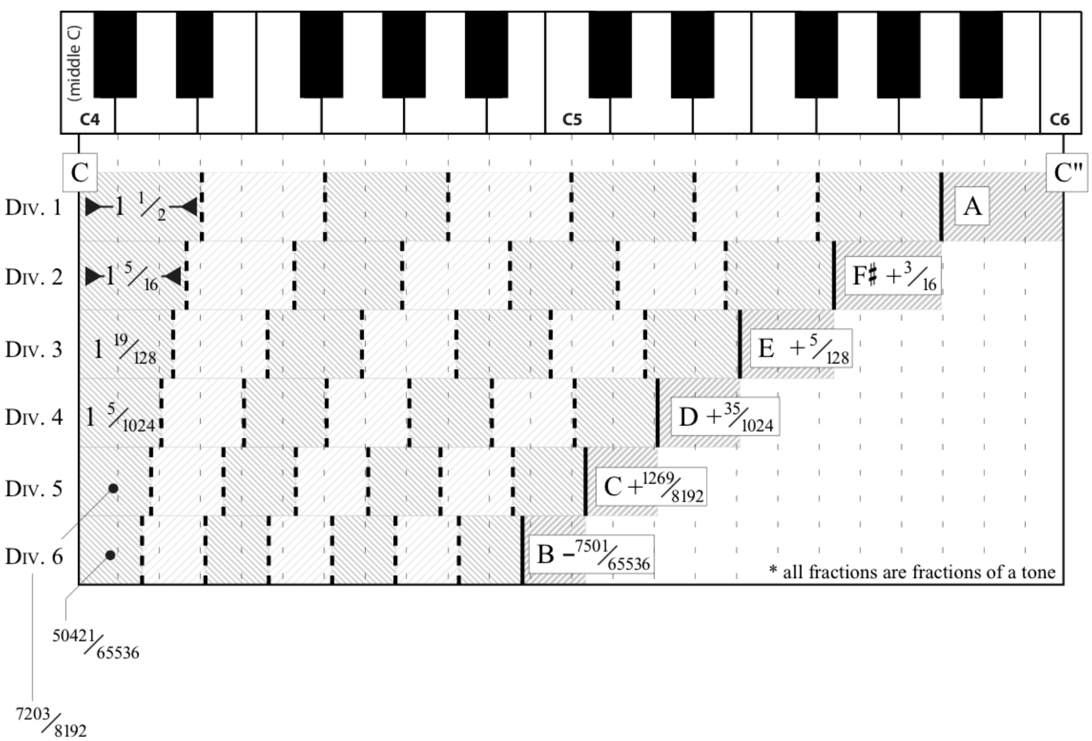


Seventh Divisions





Eighth Divisions



APPENDIX B

RECITAL PROGRAM

THE BOB COLE CONSERVATORY OF MUSIC AT
CALIFORNIA STATE UNIVERSITY, LONG BEACH AND THE COLLEGE OF THE ARTS PROUDLY PRESENT:

SETH SHAFER

COMPOSER

WITH THE UNIVERSITY BRASS ENSEMBLE,
THE BELMONT STRING QUARTET,
THE LAPTOP ENSEMBLE,
ANNA MONSMA, AND
DAVID GOYETTE

IN A GRADUATE RECITAL

MONDAY, MARCH 19, 2012 // 8:00PM

GERALD R. DANIEL RECITAL HALL

PLEASE SILENCE ALL ELECTRONIC MOBILE DEVICES.

PROGRAM

Pulsar (2009)

David Goyette–trombone

Quasar (2009–2010)

The University Brass Ensemble, Dr. Martin Herman–conductor

Music for Knobs I (2011)

The Laptop Ensemble: David Landon, Nathanael Tronerud,
Therisse Martinez, and Zach Lovitch–knob

The 20th Century Motor Company (2010)

The Belmont String Quartet: Jennifer Tuinenga and Joanna Alpizar–violins,
Corinne Olsen–viola, Chris McCarthy–cello

INTERMISSION

Anomaly (2011)

Anna Monsma–flute,
Melissa Carrington–oboe, Monica Cummins–clarinet,
Ramon Villanueva–horn, Daniel Ridgway–trombone,
Philip Glenn–violin, Andy Zacharias–double bass,
Seth Shafer–processing, Jeffery de Seriere–conductor

Bio-Glow (2009)

Kevin Brown, Nick Gilroy, Andrew McAfee, Jake Nilson,
and Jazper Saldana–percussion, Seth Shafer–conductor

Like a Spear from the Earth (2011)

The Belmont String Quartet: Jennifer Tuinenga and Joanna Alpizar–violins,
Corinne Olsen–viola, Chris McCarthy–cello

NOTES

PULSAR for trombone and computer was written to explore some of the sonic possibilities of live delay. Four different types of delay are used to evoke contrasting aspects of interstellar space.

2

QUASAR grew out of a previous piece, *Pulsar*. Instead of using signal processing to create elaborate delays, three brass quintets mimic and distort each other's sound.

MUSIC FOR KNOBS I is one of a series of short pieces that attempt to comment on the human-computer relationship by reducing the total interaction to a single knob. Each performer controls a 300º knob that acts as the sole vehicle for creating sound. The goal of this piece is to explore the simple gesture of turning a knob and to exploit those turns to their maximum musicality.

THE 20TH CENTURY MOTOR COMPANY Named after one of the many fictional enterprises from Ayn Rand's *Atlas Shrugged*, this piece explores the relentless determination of the human mind in the face of opposition. Like other industries in the novel, the Motor Company's story is one of past greatness, squandered potential, and recovered genius.

ANOMALY According to the scientific philosopher Thomas Kuhn, there are two types of science: normal and revolutionary. In the realm of normal science, there are generally accepted methods of defining and solving problems. The world can be categorized and sorted, theories can be refined and upheld, and both classrooms and laboratories can rest assured that their conclusions will consistently verify their hypotheses. And then there is revolutionary science. This is the study by which unexpected things occur, the practice where laws of gravity stem from apples, mathematics describes a cosmic ballet in the heavens, and enormous amounts of energy are released when atoms split in half. The difference between the two sciences, the marker that defines the transition from the normal to the revolutionary, the event that demands a new paradigm, is an anomaly.

BIO-GLOW While watching the waves curl and crash at Seal Beach one fall night, a blue-green explosion of light in the water caught my eye. The bioluminescent reaction of single-cell algae agitated by the breaking waves created a kaleidoscope of glowing patterns, blooming and disappearing in the froth. A fish darted near the surface in spastic S-curves, the light-trail betraying its secret path. Above the ocean, a half-moon wandered in and out of cloud thickets, sometimes casting its silvery light on the crests, and other times abandoning the water to its dark ways. And then, just as the tide began to push forward again, to heave its heavy shoulder once more into the sand: Look! the sea and everything in it glows.

LIKE A SPEAR FROM THE EARTH In 1977, NASA launched a space probe whose mission was to study the outer solar system and beyond. Over 30 years later, Voyager 1 is the man-made object farthest from Earth, and one of a handful of other such objects close to leaving our solar system and entering the vast desert of interstellar space. Equipped with an unusual time capsule with artifacts of intelligent life from earth, Voyager 1 is hurtling at over 38,600 miles per hour into the big, empty unknown. In the great darkness, it is a beacon of light thrown by mankind.

This recital is presented in partial
fulfillment of the requirements for the

MASTER OF MUSIC DEGREE

with an option in
Composition

Seth Shafer
is a student of

Dr. Martin Herman

and has studied with

Dr. Alan Shockley
Dr. Robin Cox
and
Dr. Adriana Verdie

A special thank you: to my wife, Abigail, for her loving encouragement, my parents for their support, my teachers for their guidance, my fellow graduate students and friends, Zach Lovitch and Andy Zacharias, for sharpening me, Dr. Herman and his vision for the Laptop Ensemble, Dr. Rob Freer and his preparation of The University Brass Ensemble, Rychard Cooper, David Goyette, David MacEwan, and Mike St. Claire for their technical assistance, and all of tonight's performers for their talent and generosity, without which this would not have been possible.

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